

Every definition of analysis is a sentence about sets and functions. *Logic and Proof* supplied the grammar of such sentences: the connectives, the quantifiers, and the prose that carries their rules. This volume supplies the nouns: sets and functions, relations and orders, the natural numbers and induction, and the sizes of infinite sets.

Who it is for. The reader we have in mind has worked through *Logic and Proof*, or has its habits from elsewhere: reading a statement with its quantifiers in place, proving a “for every” by taking an arbitrary element, naming a witness before using it. The chapters point back to *Logic and Proof* where they use one of its rules, so a reader who is unsure of a step can look it up.

What it covers.

Chapter	What it does
1	Sets and the working rules that say which sets exist, Russell’s paradox, families of sets, products and relations, functions and their inverses, images and preimages
2	The arithmetic of $\mathbb{N}$ and $\mathbb{Z}$ from induction, well-ordering, induction as a craft, definition by recursion, equivalence relations and partitions, partial orders
3	Finite and countable sets, the axiom of choice, diagonal arguments and Cantor’s theorem, the Schröder–Bernstein theorem, infinite sets
4	Zorn’s lemma

Chapters 1–3 are written; Chapter 4 is not yet. The real numbers are used throughout as familiar objects, with their school arithmetic and order, and every use of a school fact is named where it occurs. *Analysis on the Real Line* turns those facts into consequences of axioms, and *Number Systems: From $\mathbb{N}$ to $\mathbb{R}$* builds the numbers out of sets.

How to read it. Read the chapters in order: each uses the ones before it. Every section has Quick checks with folded answers; answer each one before you unfold it. The exercises of a section come in three groups, A to check understanding, B for practice and C to go deeper, and the last section of each chapter adds R items that reach back to earlier chapters. A chapter’s opening page lists what it needs, with links, and the time it takes.