

Every definition of analysis is a sentence about sets and functions. *Logic and Proof* supplied the grammar of such sentences: the connectives, the quantifiers, and the prose that carries their rules. This volume supplies the nouns, and above all the two that every other one is made of: sets and functions. Around them it builds the natural numbers and induction, the sizes of infinite sets and their arithmetic, relations and orders, and the ways a function is built from others.

Who it is for. The reader we have in mind has worked through *Logic and Proof*, or has its habits from elsewhere: reading a statement with its quantifiers in place, proving a “for every” by taking an arbitrary element, naming a witness before using it. The chapters point back to *Logic and Proof* where they use one of its rules, so a reader who is unsure of a step can look it up.

What it covers.

Chapter	What it does
1	Proofs from definitions: sets and functions. Proofs straight from definitions (direct proof, cases, counterexamples) on even, odd, divides and prime; sets and the working rules that say which sets exist, Russell’s paradox, families of sets, products and relations, functions and their inverses, images and preimages, each section with its method of proof
2	The arithmetic of $\mathbb{N}$ and $\mathbb{Z}$ from induction, well-ordering, induction as a craft, definition by recursion, contrapositive and contradiction, infinitely many primes, equinumerous and finite sets and the pigeonhole principle
3	Countable sets, the axiom of choice, diagonal arguments and Cantor’s theorem, the Schröder–Bernstein theorem, infinite sets; cardinal arithmetic: sums, products and powers of sizes, $ \mathbb{R} = \mathcal{P}(\mathbb{N}) $, $\mathbb{R}^2$ and $\mathbb{N}^{\mathbb{N}}$ as large as $\mathbb{R}$ (Section 3.6)
4	Relations: properties of relations decided one at a time, equivalence relations and partitions, the integers modulo n , well-definedness, partial orders, maximal and greatest elements, least upper bounds
5	Building functions: restriction, extension and gluing, indicator functions; fibers and the factorization of every function through a quotient; binary operations and their identities and inverses; permutations, iteration, orbits and cycles

Chapter	What it does
6	Zorn's lemma, stated without proof, and the maximality template: the axiom of choice from Zorn's lemma, any two sets are comparable in size, every partial order extends to a total order

The real numbers are used throughout as familiar objects, with their school arithmetic and order, and every use of a school fact is named where it occurs. *Analysis on the Real Line* turns those facts into consequences of axioms, and *Number Systems: From $\mathbb{N}$ to $\mathbb{R}$* builds the numbers out of sets.

How to read it. Read the chapters in order: each uses the ones before it. Every section has Quick checks with folded answers; answer each one before you unfold it. When a section uses a method of proof for the first time, it sets the method out in a box marked *Method*, with a template, when to use it and the usual failure; later sections cite the box instead of explaining the method again, and each chapter's opening page lists its boxes under Named moves. The exercises of a section come in three groups, A to check understanding, B for practice and C to go deeper, and the last section of each chapter adds R items that reach back to earlier chapters. A chapter's opening page lists what it needs, with links, and the time it takes: about 160 hours for the whole volume on the core track, and about 225 hours for everything.