

Number Systems: From $\mathbb{N}$ to $\mathbb{R}$ ended with the real numbers as a complete ordered field. *Analysis on the Real Line* starts from the axioms of such a field and proves its first limit theorems at once. This volume supplies the working tools that analysis uses on every page without stopping to prove them: inequalities and finite sums, real functions and their graphs, coordinates and vectors in $\mathbb{R}^n$, lines, convex sets and isometries, the complex numbers and the plane they form, and distances in the general form of a metric.

Who it is for. The reader we have in mind has worked through *Logic and Proof*, *Sets and Functions* and the main line of *Number Systems: From $\mathbb{N}$ to $\mathbb{R}$* , or has their habits from elsewhere. Much of the material is familiar in outline from school; what is new is that every rule is proved from the axioms of a complete ordered field, and that nothing is used before it is proved.

What it covers. The volume is being written. Its finished chapter is:

Chapter	Title	What it does
1	Complex Numbers	$\mathbb{C}$ as pairs of real numbers, proved a field containing $\mathbb{R}$; conjugate, modulus and the triangle inequality; square roots and the quadratic formula; why $\mathbb{C}$ has no order; polar form, De Moivre's formula and the roots of unity

Six chapters are planned around it, in this reading order: *Inequalities, Finite Sums, and Real Powers*; *Real Functions and Their Graphs*; *Coordinates and Vectors in $\mathbb{R}^n$* ; *Lines, Convexity, and Isometries of $\mathbb{R}^n$* ; then *Complex Numbers*; then *Rotations, Angles, and the Complex Plane* and *Metrics and Norms*. When they are added, the chapters will be renumbered in that order.

How to read it. Every section has Quick checks with folded answers; answer each one before you unfold it. The exercises of a section come in three groups, A to check understanding, B for practice and C to go deeper, and the last section of each chapter adds R items that reach back to earlier chapters. A chapter's opening page lists what it needs, with links, and the time it takes. Second-reading folds hold edge cases and longer justifications. They can stay closed on a first reading, but they are not optional, since later proofs may use what they contain.

One borrowed fact. This volume does not define cosine, sine or π : defining them needs limits and power series, which belong to analysis. Section 1.5 states five facts about them, numbered (T1)–(T5), as an assumption, and *Analysis on the Real Line*, Chapter 10 proves them. They are used only for polar form and for the roots of complex numbers in Sections 1.5 and 1.6, and each proof that uses one names it.

Time. Chapter 1 takes about 25 hours on the core track and about 35 hours for everything.

What comes next. *Analysis on the Real Line* uses this volume from its second chapter on. *Algebraic Preliminaries* takes the complex numbers of Chapter 1 as its main example and builds fields, groups and polynomials on them, ending with the Fundamental Theorem of Algebra. The linear algebra volumes, beginning with *Spaces and Maps*, use the complex numbers throughout.